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Title: The Sm2 ratio for evaluating neutron multiplicity models.

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The Sm_2 ratio for evaluating neutron multiplicity models

Application Note

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Introduction

Typically when a model of a neutron emitting system made the accuracy of the model is validated against a measurement. Typically the total count rate of a modelled detector is compared with the count rate of a measurement. Unfortunately, small uncertainties in the measured position of the detector contribute to the uncertainty in the modeled detector and can hide discrepancies in the modelling of the object. This is because detector placement is important because slight changes in the location of the detector change the efficiency of the detector.

If the model of interest is of multiplying medium, such as special nuclear material, then doubles can be measured with a multiplicity detector. The information from the doubles can be used for a better evaluation of the model where the precision of the detector placement is not as critical to the overall validation of the model of the nuclear material.

Theory

In the Hage Cifarelli formulism the singles and doubles rates are used to estimate neutron multiplication and spontaneous fission rate [1]. The singles rate is given by

$$R_1 = \varepsilon M_L \overline{\nu_{S1}} F_S \tag{1}$$

and the doubles rate is given by

$$R_2 = \varepsilon^2 M_L{}^2 \left(\overline{\nu_{S2}} + \frac{M_L - 1}{\overline{\nu_{I1}} - 1} \overline{\nu_{S1}}\,\overline{\nu_{I2}} \right) F_S \,, \tag{2}$$

where the parameters in eqn's 1 and 2 are defined in Table 1.

Table 1. Definition of constants used in eqn's 1 and 2.

Symbol	Definition
R_1	Singles rate
R_2	Doubles rate
ε	Total detector efficiency
M_L	Leakage multiplication
F_S	Spontaneous fission rate
$\overline{\nu_{I1}}$	First reduced moment of the induced fission multiplicity distribution. Also the average number of neutrons emitted per induced fission.
$\overline{\nu_{I2}}$	Second reduced moment of the induced fission multiplicity distribution.
$\overline{\nu_{S1}}$	First reduced moment of the spontaneous fission multiplicity distribution. Also the average number of neutrons emitted per spontaneous fission.
$\overline{\nu_{S2}}$	Second reduced moment of the spontaneous fission multiplicity distribution.

The single and double equations can be easily interpreted as

$$\textit{Measurement} = \textit{Detector Response} \times \textit{Material Properties}, \tag{3}$$

where the *Measurements* corresponds to measured properties, such as R_1 or R_2, the *Detector Response* corresponds to either ε or ε^2, and *Material Properties* are those parameters pertaining to the neutron source, which are multiplicity constants, leakage multiplication, and fission rate.

By taking the ratio

$$Sm_2 = \frac{R_2}{R_1^2} = \frac{\overline{\nu_{S2}} + \frac{M_L - 1}{\overline{\nu_{I1}} - 1}\overline{\nu_{S1}}\,\overline{\nu_{I2}}}{\overline{\nu_{S1}}^2 F_S} \tag{4}$$

the detector response cancels out and a ratio that only contains material properties is left.

This parameter is independent of detector response and hence when comparing modeling results of detectors with actual measurements this ratio should be equal. If they are not equal then most likely there is a discrepancy in the model.

One thing to note is that if the multiplication in a model is off, then both the singles rate and the Sm_2 value will be off in the same direction. i.e. both R_1 and Sm_2 value will be both high or low. However, if the source strength is off then R_1 will be off in the opposite direction that the Sm_2 value is. E.g. if the count rate is high and the Sm2 value is low, then a modification to the source strength term is most likely at fault, assuming the multiplicity values are correct.

The uncertainty for Sm_2 is

$$\sigma_{Sm_2} = Sm_2\sqrt{4\left(\frac{\sigma_{R_1}}{R_1}\right)^2 + \left(\frac{\sigma_{R_2}}{R_2}\right)^2}\ . \tag{5}$$

For comparison another parameter will be evaluated, R_{2f}. R_{2f} is given by

$$R_{2f} = \frac{R_2}{R_1} = \varepsilon M\,\frac{\overline{\nu_{S2}} + \frac{M_L - 1}{\overline{\nu_{I1}} - 1}\overline{\nu_{S1}}\,\overline{\nu_{I2}}}{\overline{\nu_{S1}}}\ . \tag{6}$$

In R_{2f}, the detector response (ε) is still present and doesn't represent the modelling of the object alone.

The uncertainty for R_{2f} is

$$\sigma_{R_{2f}} = R_{2f}\sqrt{\left(\frac{\sigma_{R_1}}{R_1}\right)^2 + \left(\frac{\sigma_{R_2}}{R_2}\right)^2} \tag{7}$$

Proof of Concept

To test this method, nine point-model monte-carlo simulations were performed. Nine separate sets of synthetic data were generated where the material properties, i.e. the multiplicity constants, multiplication, and spontaneous fission rate were kept constant. The detector response was changed by varying efficiency. The object modeled was the Beryllium Reflected Plutonium Sphere, or BeRP ball. The BeRP ball is 4.5 kg's of alpha-phase plutonium with 6% ^{240}Pu[2]. The modeled detector was of the Los Alamos National Laboratory's neutron pod, or nPod and the dimensions of the nPod are 43.2 cm wide by 41.9 cm tall and the simulated time for all simulations was 5 minutes. Nominally, the nPod detector's efficiency at 50 cm away from the BeRP ball is 1%. The efficiency was changed to correspond to nine different distances of the nPod away from the BeRP ball.

The total efficiency of a detector is given by

$$\varepsilon_{Total} = \frac{\Omega}{4\pi}\varepsilon_{Intrinsic} \tag{8}$$

where Ω is the solid angle of the detector. For a rectangular plate on a sphere the solid angle is given by

$$\Omega(a,b,d) = 4\arccos\sqrt{\frac{1+\alpha^2+\beta^2}{\left(1+\alpha^2\right)\left(1+\beta^2\right)}} \tag{9}$$

where $\alpha = \frac{a}{2d}$ and $\beta = \frac{b}{2d}$ [3].

For each of the nine simulations the singles and doubles rates and uncertainty were calculated using a random sampling method [4]. The efficiencies, the single, and doubles rates for the simulations at various distances are given in Table 2 and the Sm_2 and R_{2f} values are given in Table 3.

Table 2. The single and double rates of nine monte-carlo simulations of the BeRP ball and an nPod at various distances.

d (cm)	$\frac{\Omega}{4\pi}$	ε_{Total}	R_1	R_2
48.0	0.05249	0.01072	10,143.6 ± 7.2	1,634.5 ± 21.4
48.5	0.05158	0.01053	9,955.1 ± 6.9	1,586.9 ± 20.0
49.0	0.05069	0.01035	9,791.6 ± 6.8	1,543.8 ± 21.5
49.5	0.04982	0.01017	9,621.0 ± 6.6	1,463.6 ± 20.7
50.0	0.04898	0.01000	9,451.7 ± 7.0	1,413.4 ± 19.7
50.5	0.04815	0.00983	9,299.2 ± 6.6	1,369.5 ± 18.6
51.0	0.04734	0.00967	9,145.1 ± 6.7	1,315.1 ± 19.6
51.5	0.04656	0.00951	8,995.1 ± 6.5	1,282.1 ± 18.5
52.0	0.04579	0.00935	8,845.3 ± 6.4	1,243.3 ± 17.6

Table 3. The R_{2f} and Sm_2 values for the nine monte-carlo simulations of the BeRP ball and an nPod at various distances.

d (cm)	$\frac{\Omega}{4\pi}$	ε_{Total}	R_{2f}			Sm_2		
48.0	0.05249	0.01072	0.1611	±	0.0021	1.59E-05	±	0.021E-05
48.5	0.05158	0.01053	0.1594	±	0.0020	1.60E-05	±	0.020E-05
49.0	0.05069	0.01035	0.1577	±	0.0022	1.61E-05	±	0.023E-05
49.5	0.04982	0.01017	0.1521	±	0.0022	1.58E-05	±	0.022E-05
50.0	0.04898	0.01000	0.1495	±	0.0021	1.58E-05	±	0.022E-05
50.5	0.04815	0.00983	0.1473	±	0.0020	1.58E-05	±	0.022E-05
51.0	0.04734	0.00967	0.1438	±	0.0021	1.57E-05	±	0.024E-05
51.5	0.04656	0.00951	0.1425	±	0.0021	1.58E-05	±	0.023E-05
52.0	0.04579	0.00935	0.1406	±	0.0020	1.59E-05	±	0.023E-05

It's easier to understand the results when graphed and so in Figure 1 and Figure 2 are the R_{2f} and Sm_2 values plotted as a function of nPod distance from the BeRP ball. It can be seen that over a broad range the Sm_2 value is constant and independent of the detector placement. The R_{2f} value, however, shows significant variation when the efficiency changes by a detector placement more than 1.5 cm.

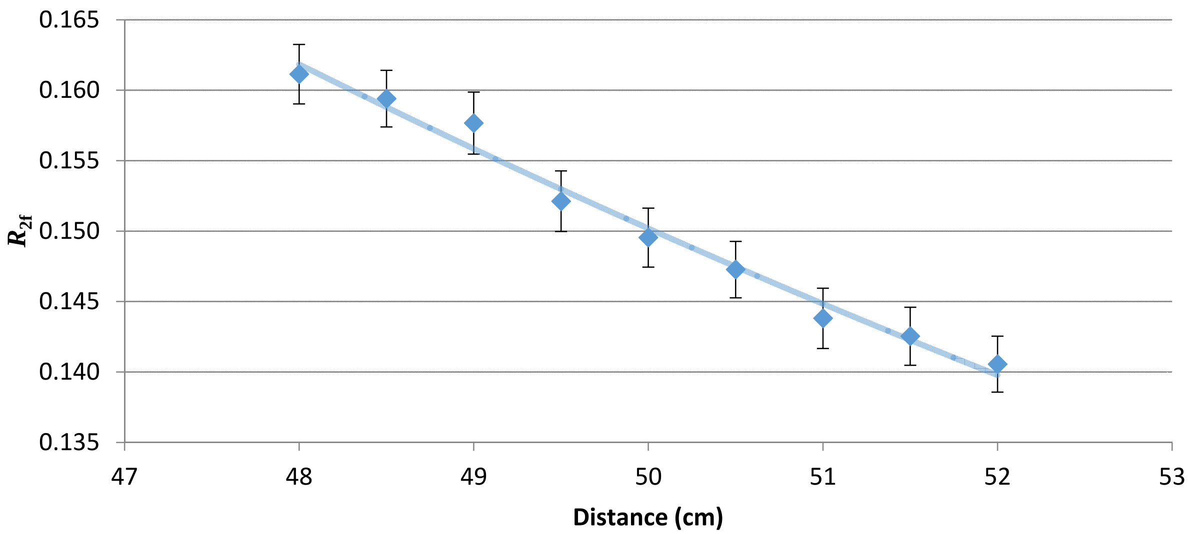

Figure 1. A plot of the R_{2f} values as a function of distance for the nine simulations of the BeRP ball and the nPod.

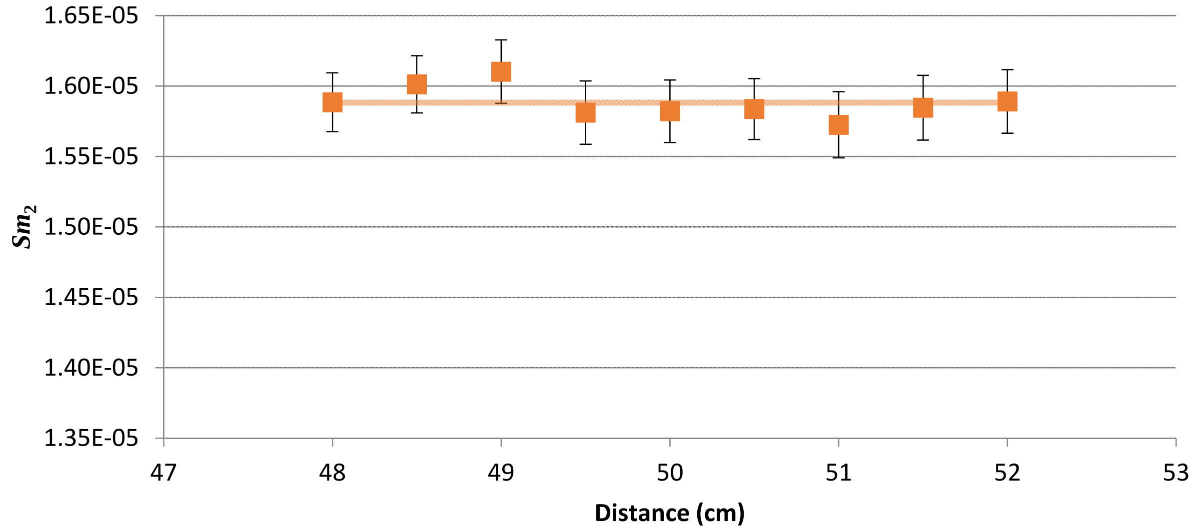

Figure 2. A plot of the Sm_2 values as a function of distance for the nine simulations of the BeRP ball and the nPod.

Conclusion

This paper has shown that in a relatively ideal world that the Sm2 ratio is a parameter that can separate source material properties from detector response. This method uses the doubles of a multiplicity counter to eliminate the detector response in a measurement. Thus, the uncertainties in detector placement in relationship to the nuclear material being modelled are reduced and a more accurate assessment of how well the nuclear material was modelled is emphasized.

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[4] Cutler, T.E., Smith-Nelson, M.A., Hutchinson, J.D. "Deciphering the Binning Method Uncertainty in Neutron Multiplicity Measurements." Los Alamos National Laboratory, LA-UR-14-23374. (May 2014).